

Probability and Random Processes

EES 315

Asst. Prof. Dr. Prapun Sukksompong

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6.3 Bernoulli Trials



Office Hours:

Check Google Calendar on the course website.

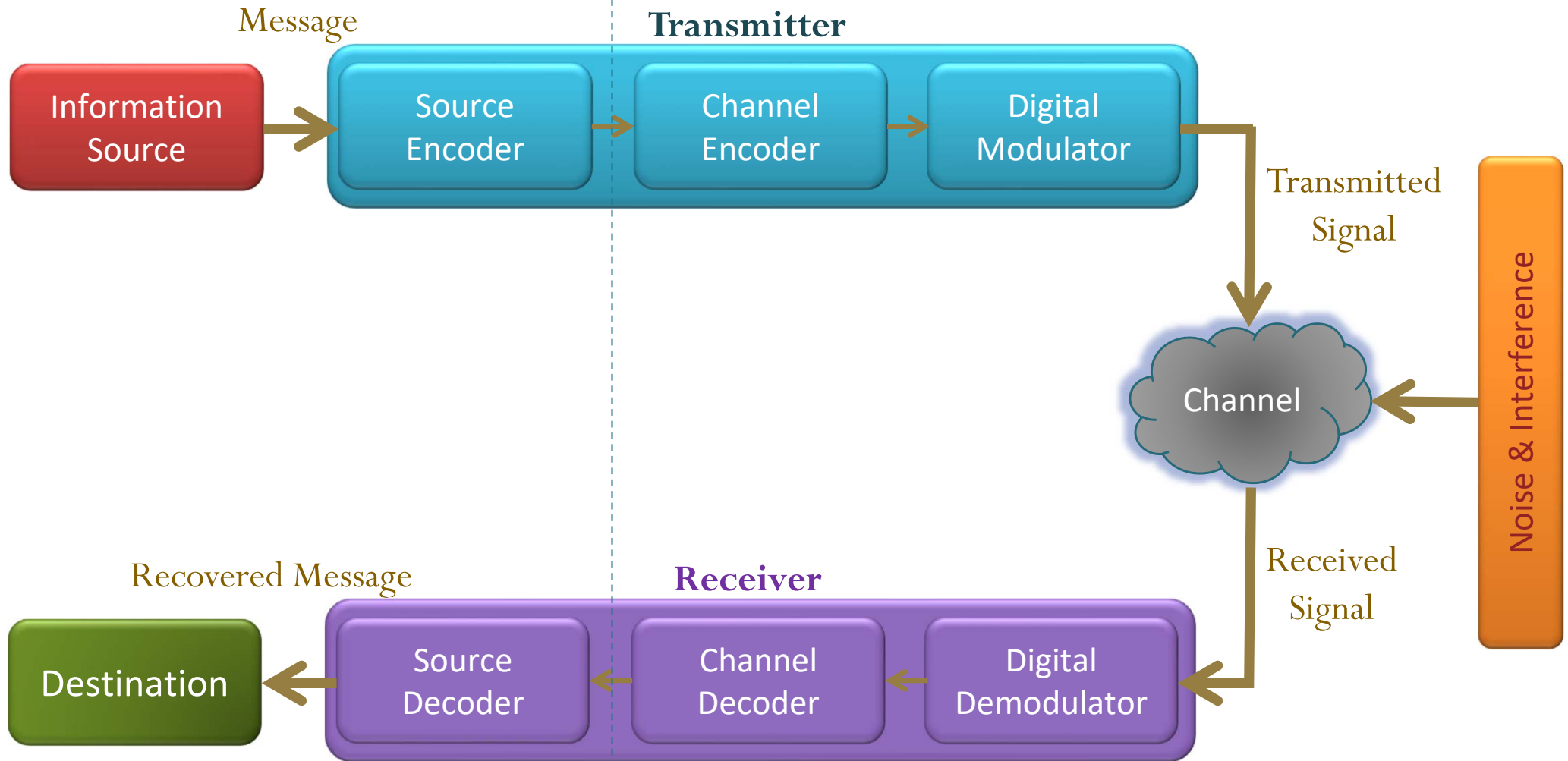
Dr.Prapun's Office:

6th floor of Sirindhralai building,
BKD

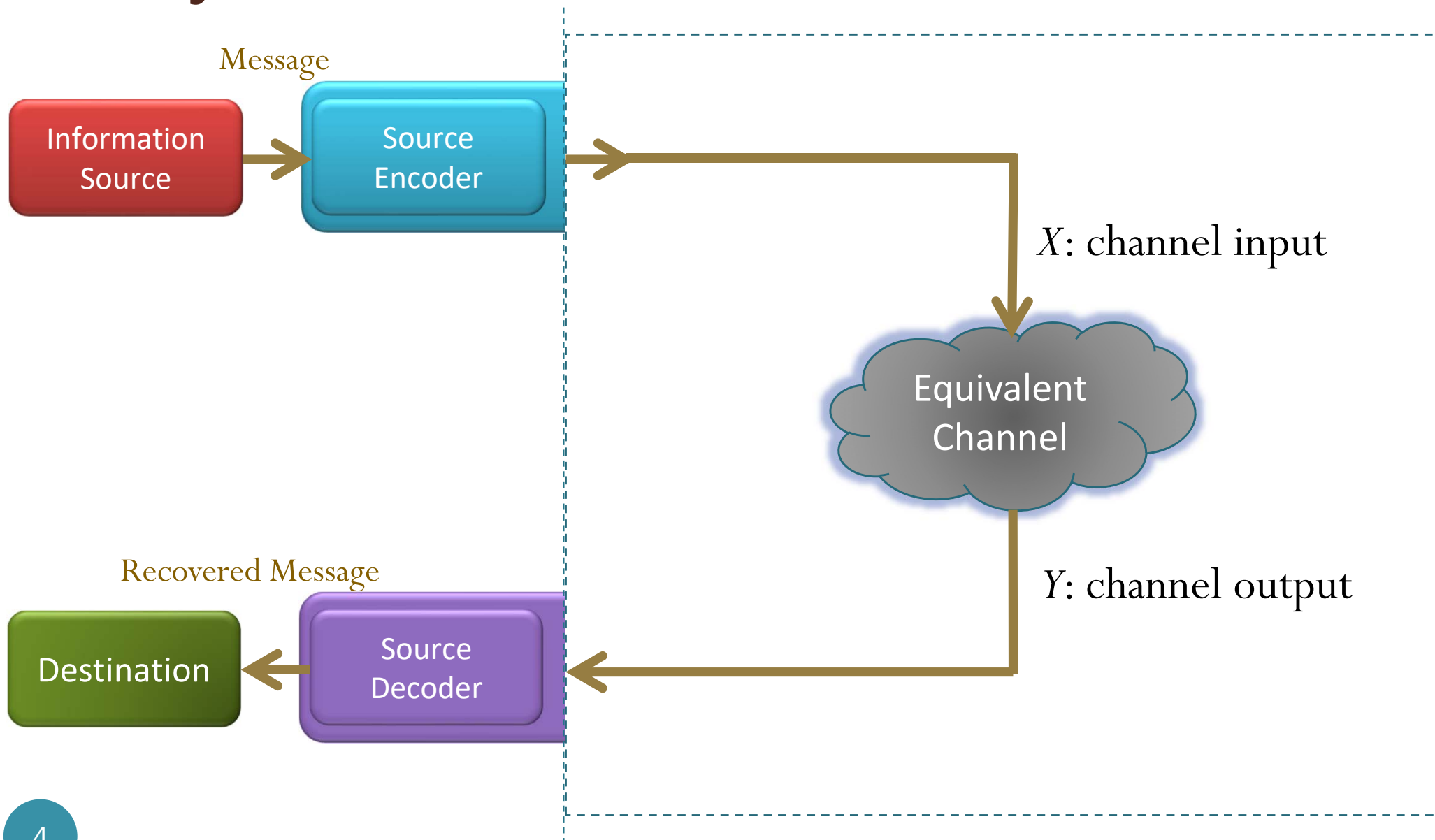
[Ex. 6.57] $1 - \left(1 - \frac{1}{n}\right)^n$

n	$1 - \left(1 - \frac{1}{n}\right)^n$
1	1
10	0.651322
64	0.635013
65	0.634969
100	0.633968
1000	0.632305
10000	0.632139

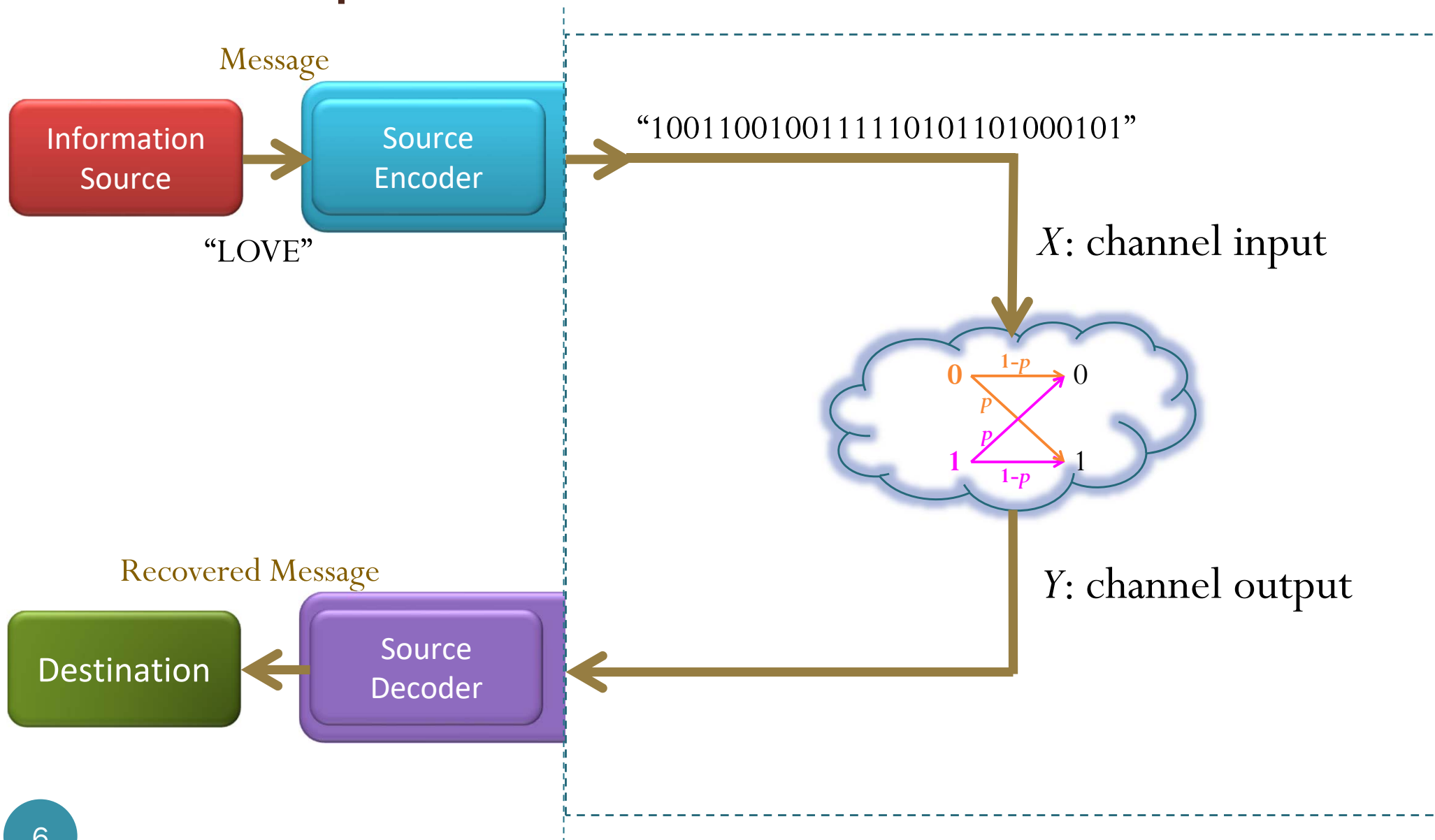
Elements of digital commu. sys.



System considered



Example: ASCII Encoder and BSC



The ASCII Coded Character Set

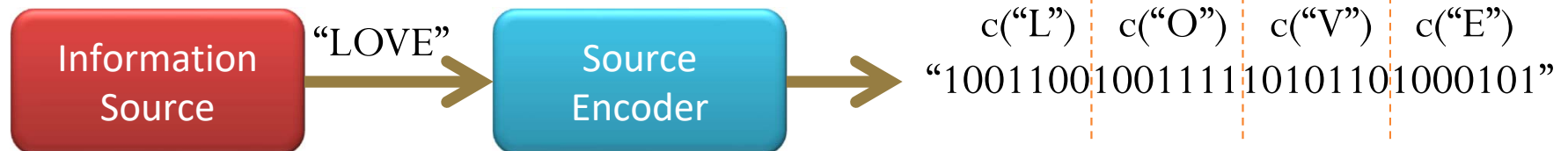
				6	0	0	0	0	1	1	1	1							
<i>Bit</i>				5	0	0	1	1	0	0	1	1							
<i>Number</i>				4	0	1	0	1	0	1	0	1							
				1st	0	1	2	3	4	5	6	7							
3	2	1	0	Hex															
				2nd															
0	0	0	0	0	0	NUL	16	DLE	32	SP	48	0	64	@	80	P	96	112	p
0	0	0	1	1	1	SOH		DC1		!	1	A		Q		a		q	
0	0	1	0	2	2	STX		DC2		"	2	B		R		b		r	
0	0	1	1	3	3	ETX		DC3		#	3	C		S		c		s	
0	1	0	0	4	4	EOT		DC4		\$	4	D		T		d		t	
0	1	0	1	5	5	ENQ		NAK		%	5	E		U		e		u	
0	1	1	0	6	6	ACK		SYN		&	6	F		V		f		v	
0	1	1	1	7	7	BEL		ETB		'	7	G		W		g		w	
1	0	0	0	8	8	BS		CAN		(8	H		X		h		x	
1	0	0	1	9	9	HT		EM)	9	I		Y		i		y	
1	0	1	0	A	A	LF		SUB		*	:	J		Z		j		z	
1	0	1	1	B	B	VT		ESC		+	;	K		[k		{	
1	1	0	0	C	C	FF		FS		,	<	L		\		l			
1	1	0	1	D	D	CR		GS		-	=	M]		m		}	
1	1	1	0	E	E	SO		RS		.	>	N		^		n		~	
1	1	1	1	F	F	SI		US		/	?	O		_		o		DEL	

Example: ASCII Encoder

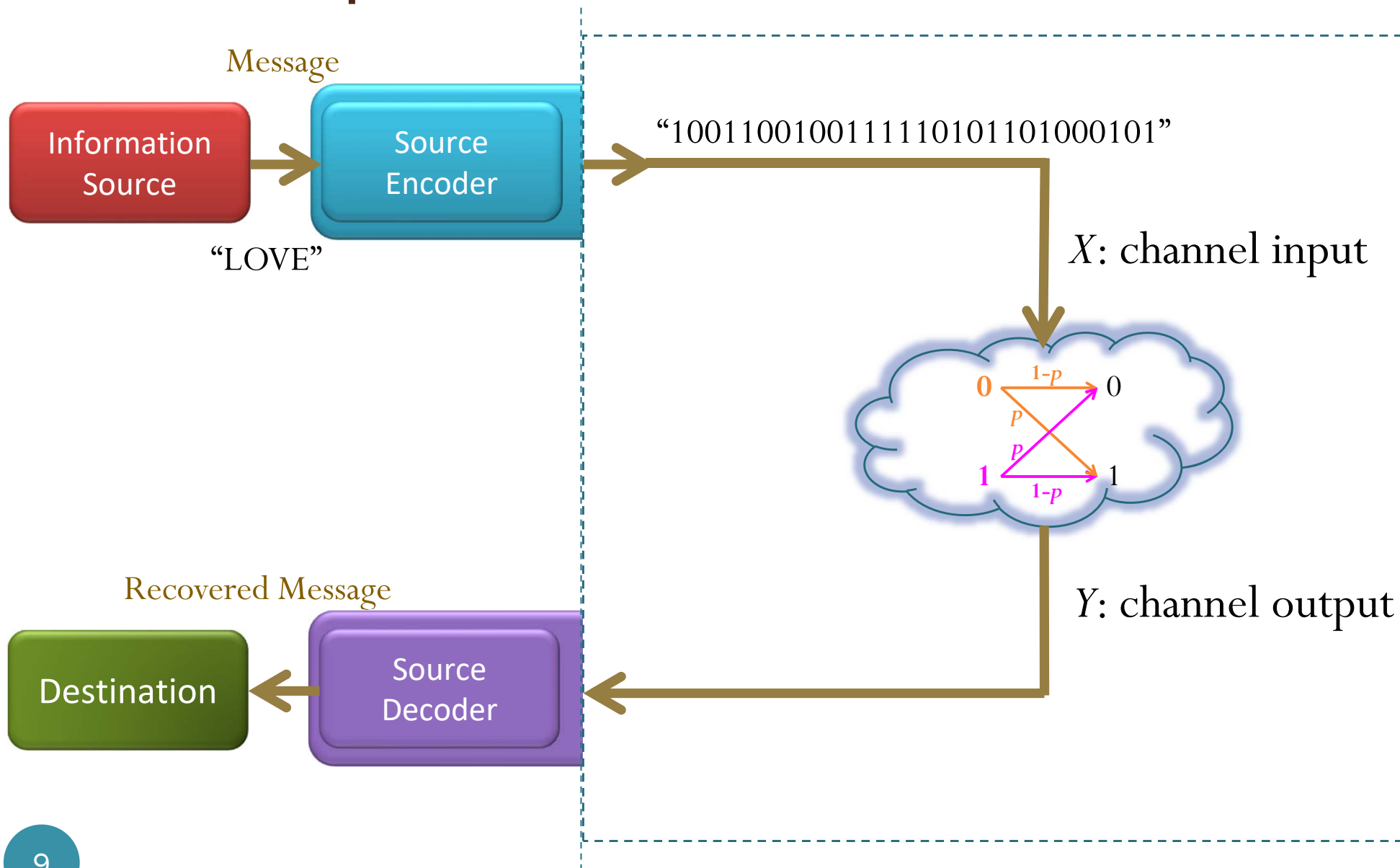
Character	Codeword
:	
E	1000101
:	
L	1001100
:	
O	1001111
:	
V	1010110
:	

MATLAB:

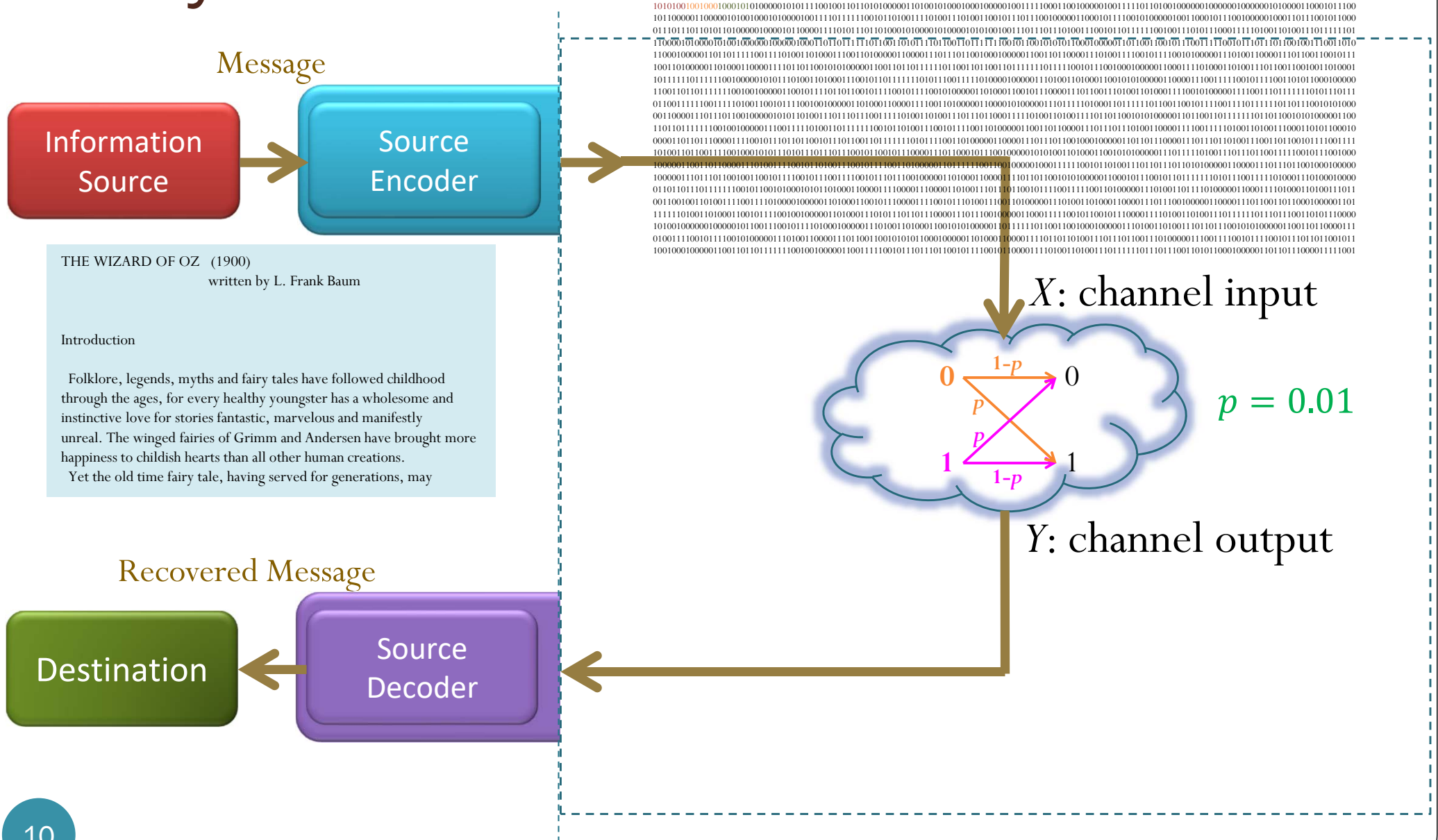
```
>> M = 'LOVE';  
>> X = dec2bin(M, 7);  
>> X = reshape(X', 1, numel(X))  
X =  
1001100100111110101101000101
```



Example: ASCII Encoder and BSC



System considered



Results

THE WIZARD OF OZ (1900)
written by L. Frank Baum

Introduction

Folklore, legends, myths and fairy tales have followed childhood through the ages, for every healthy youngster has a wholesome and instinctive love for stories fantastic, marvelous and manifestly unreal. The winged fairies of Grimm and Andersen have brought more happiness to childish hearts than all other human creations. Yet the old time fairy tale, having served for generations, may



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```
101010010010001000101010000010101110010011011010100000110100101000100001000001111100011001000001001111011010010000001000000100000110001011100
1011000001100000101001000101000010011110111100101101001111010011101001100011001110110010000011000101111001000001000101110010011000
01110111011010110110000010000101100001110101110110000101000010100001010010011101110100111001011110010011100101110010111001011100101110010111001011001011001010
1100001010000101001000000100000100011011011110110011010111101001100101011000100000110110011001011001111001011101101100100110011010
1100010000011011110011110011101000111001101000001100011101101111101001101100110111110111100101110010000011010011000011101001100001110100110010111
100110100000110100011000011110110110010101000001100110110111110100110110011011111011110010111001000010000110001110100011010011101100110010011010011
1011110111100100000101011101001101000111001011111010111001111010000100000111010011010001100101100001100011110010111001010100010000
110011011011111001001000001100101111011011001011100101110010100000110100011001011100001110100111010011100011110010100000111001110101110101110111
01100111110011110100110010111001001000001101000110000111001101000001100001101011110100011011111010011001011100111011110101110010101000
10011000011101110010000001010100111011100111010011101001100111101001100111101001100111101001100111101011001010100001100
1101011111001001000001110011110100110111100101101001111001011010011100101110010110000011001101000011100110100110000110010100001100
00011011011000011100101110101001011101001011111010111001101000001100001110110110010001000001101101100001110101101001110011010010111001111
1010011010011100100010101110101110110010110010111000011101100010111001000001010001101000110010101000001110111101001110111010011110010111001000
100000110011010000111010011100101100101110011010000011011110011001000001000111101011010011101011101010100000110000111011001000100000
1000001101110110010011001011100101110111001000001101000110000111101101100101010000011000101110010111110101110011110100011101000010000
011011011111001011001010001010110001100001110000111000011010011101101001011100111100101010000011101001110100001100011101000110100111011
001100100101010011100111101000010000011010001100101110000111010011101000111010011101000110011010000011010011101000110100111010011101001100101110000
11111101001010001100101110010010000011010001110101100110011000011101100100000110001110010110010111001011001011011101101100110010101110000
101001000000100000110011100101110100010000011101001101000110010100000110111101100110010001000001110100110100111010110010100000110011010000111
010011100101110010100000111010011000011011001100101011010010000111010011000011101011001011011101001110100000111001110010111001011101011001011
100100010000011001101111110010010000011001111001011101101100101110010111001011001111011101110011010110001000001101101100001111001
```



```
10101001001000100010101000001010111001001101101010000011010010100010000010111100011001000001001111011010010000010000001000001100001011100
10110000011000001100100110100001001111011110010110100111101001110100110010111011001000001100010111100100000100011011001011000
011101110110101101100001000010110000111010111011010001010000101000101010011011101110100111001011100101110010111001011100101110010111001011001010
110000101000010100100000011000010001101011110110011010111101001100101010111000000001100110010101110011110010111011011001011001011001010001000
1100010000011011011100111010011010001110011010000011000111011010010001000011001101000011101001110000111010011000011101001100101111001011100101110
1001101000001101000110000111101011001010100000110011011011100101100101111101111010011100100100000110001110100011010011010011001011001011001011001011
101111011111001000001010111010011010001110010111110101110011110100011000001110100110100011001011000001100001110011110010111100101110010101000000
1100110110111110010010001001100101110100101111001011100101000001101110100101110000111010011101001101000111001010000011100111011110101110111
011001111100101110010110010111001000001101000110000111001101000001100001110011010000110000001101111010001101111010011100101010000110010101000
0011000011011101100100000101011010011101001110011110000111001110110100011110100110100111001011001010100000110110111010110010101000001100
11011011111001001000001110011110100110111110010110100111001011100001000001100110100001010111010100011100111101001101001110010110101100010
0000110110110000111001011101100101110100101111101010110011010000011001011101100100010000011011100001101110101001110010110010111001111
101001101100111001000101011101011101100101110010000110100100001101001000011001000110100100001101001000001101001000011010010000110100100001
1000001100110100001110100111001011001011100110100000110111100110010000010001111010011000001000111010011010010000011000011011101001000101010
100000111011011001001100101110010111011100100000110111100110010000010001110100001010000011001011010111010111010111010111010111010111010111010111
0100101101111001011001010001010110010111001011101101000001101000110000111010100000110100011000011101011010111010111010111010111010111010111010111
10011001011010011100111101000100000110100011001011100001110010111010011100110100001110100110100011010011010011000011011001101011000100001101
1111100100110100011101011100100100001101000111010111000011100101110100111001011000011100101100101100111011110110110011010110110011010111000
10100100000010000010110011101000100001110100110100011001010100011011111010011001000100010110101101011010101000001100110100011101010000011000
01101101110111100101100101000101011010001100001110000111000011010011101101001011101000011000111010001100011101000110100111010110101101011010
0011001001101001110011110100010000011010001100101110000111001011101001110011010000111010011010001101001101001100011011001101011000100001101
111110010011010001110101110010010000110100011101011100001110010111001001100001100001110010110010110011101111011011001101011011010111000
1010010000001000001011001110100111010001000011010011010001100101010001101111010011001000100010110101101011010101000001100110100011101010000111000
101001000000100000101100111010001000011101001101000110010101000110111101001100100010001011101101001100100001000110101101010100000110011010000111
01001011001010000111010011000011011001100101011010010000111010110010110111010011101000001110011100101110010111001011100101110010111001011001011
```

Results

THE WIZARD OF OZ (1900)

written by L. Frank Baum

Introduction

Folklore, legends, myths and fairy tales have followed childhood through the ages, for every healthy youngster has a wholesome and instinctive love for stories fantastic, marvelous and manifestly unreal. The winged fairies of Grimm and Andersen have brought more happiness to childish hearts than all other human creations. Yet the old time fairy tale, having served for generations, may

THE WIZARD _F OZ (19009 written by L. Frank0Baum

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- The whole book which is saved in the file “OZ.txt” has 207760 characters (symbols).
- The ASCII encoded string has $207760 \times 7 = 1454320$ bits.
- The channel corrupts 14545 bits.
- This corresponds to 14108 erroneous characters (symbols).

Results

```
>> ErrorProbabilityoverBSC
biterror =
    14545
BER =
    0.010001237691842
theoretical_BER =
    0.010000000000000
symerror =
    14108
SER =
    0.067905275317674
theoretical_SER =
    0.067934652093010
```

$$\frac{14545}{1454320} \approx 0.01 \quad \leftarrow$$

$$\frac{14108}{207760} \approx 0.0679 \quad \leftarrow$$

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Results

BSC's crossover probability

$$\text{BER } p = 0.01$$

$$\frac{14545}{1454320} \approx 0.01$$

$$\frac{14108}{207760} \approx 0.0679$$

$$\text{SER} = 1 - (1 - p)^7$$

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A character (symbol) is successfully recovered if and only if none of its bits are corrupted.

Crossover probability and readability

When the first novel of the series, Harry Potter and the Philosopher's Stone (published in some countries as Harry Potter and the Sorcerer's Stone), opens, it is apparent that some significant event has taken place in the wizarding world--an event so very remarkable, even the Muggles notice signs of it. The full background to this event and to the person of Harry Potter is only revealed gradually through the series. After the introductory chapter, the book leaps forward to a time shortly before Harry Potter's eleventh birthday, and it is at this point that his magical background begins to be revealed.

Original

When the first novel of the series, Harry Pottez and the Philosopher's Stone (p5blished in some countries as Harry Potter cnd the Sorcerep's Stone), opens, it i3 apparent that soMe cignifacant event!haS taken0place in the wi~arding 7orld--ao event so `very!bemark!blu, even the Mufgles nodice signs" of it. The fuld background to this event and to the person of Harry P/tTer is only revealed gradually through th series. After the introfuctory chapter, the boo+ leaps forward to a time shortly before Harpy Potteb7s eleventh`birthday, and)t is at this poi~t that his -agikal bac{ground begins to be revealed.

$p = 0.01$ → SER ≈ 0.07

Crossover probability and readability

Human may be able to correct some (or even all) of these errors.

When the first novel of the series, *Harry Potter and the Philosopher's Stone* (published in some countries as *Harry Potter and the Sorcerer's Stone*), opens, it is apparent that some significant event has taken place in the wizarding world--an event so very remarkable, even the Muggles notice signs of it. The full background to this event and to the person of Harry Potter is only revealed gradually through the series. After the introductory chapter, the book leaps forward to a time shortly before Harry Potter's eleventh birthday, and it is at this point that his magical background begins to be revealed.

$$p = 0.01 \rightarrow \text{SER} \approx 0.07$$

Crossover probability and readability

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$$p = 0.03 \rightarrow \text{SER} \approx 0.19$$

When the first novel of the series, *Harry Potter and the Philosopher's Stone* (published in some countries as *Harry Potter and the Sorcerer's Stone*)- opens, it is apparent that a significant event has taken place in the wizarding world-, an event so very remarkable, even the Muggles notice signs of it. The full background to this event and the person of Harry Potter is only revealed gradually through the series. After the introductory chapter, the book leaps forward to a time shortly before Harry Potter's eleventh birthday and at this point their magical background begins to be revealed.

$$p = 0.02 \rightarrow \text{SER} \approx 0.13$$

Crossover probability and readability

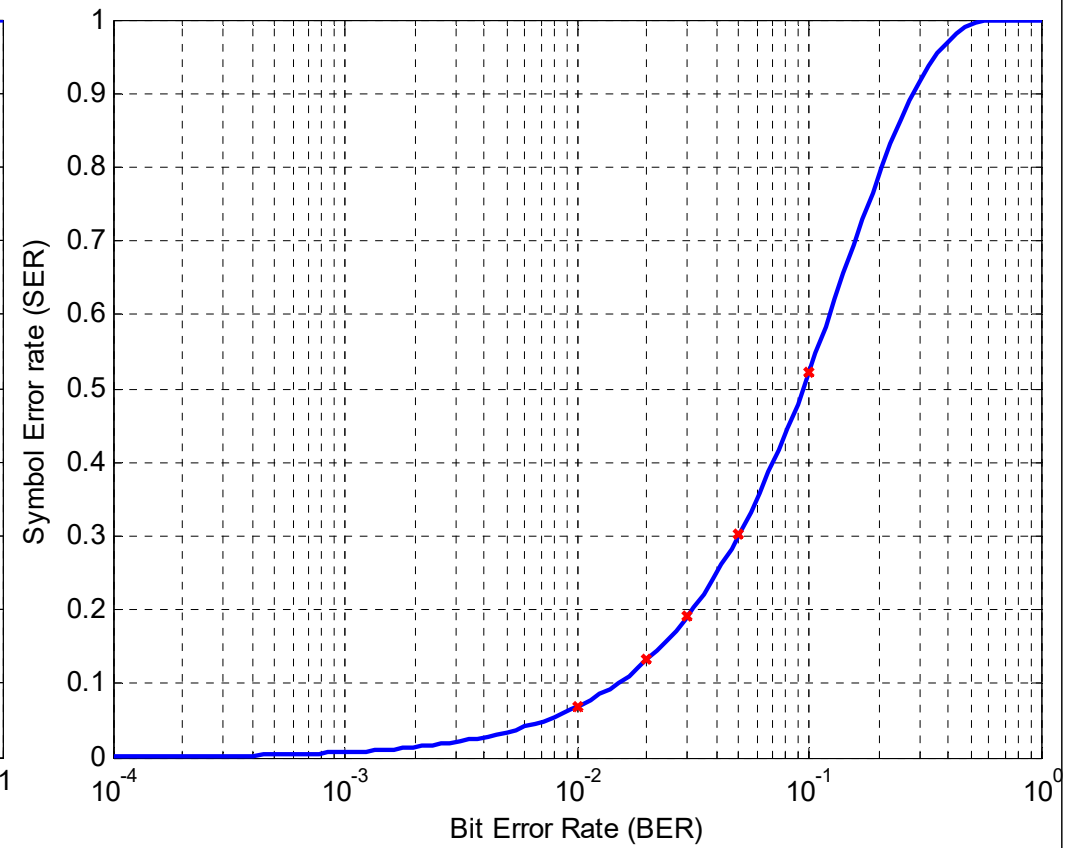
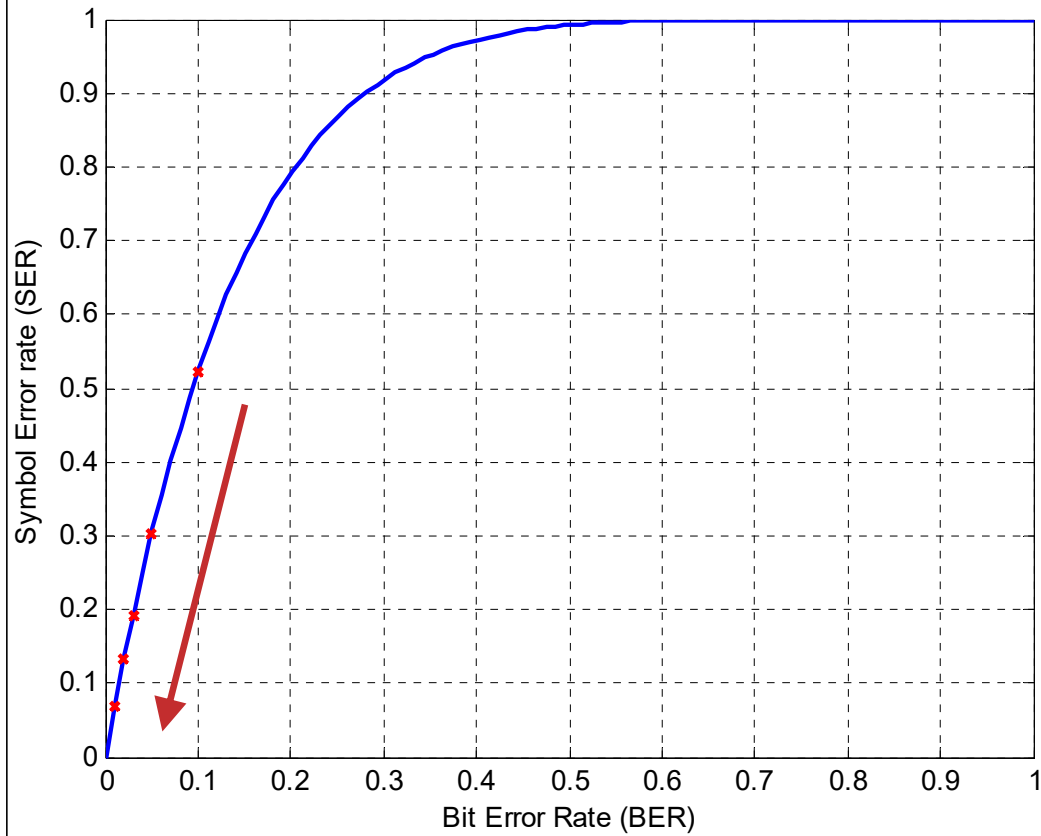
Whef th% &i2sv nkvdI"On(txE"serm-s< HaRtY Qo |p%R\$anlthe Phi\$)qop8gb'rYtoNe (puclirhed
in23/ee c uNpr9es aZ Harby!PovDdZ qnd0THA!Uorojev's Qpof'), pegsL iT is0aqazenP'Tiet` {nle
sau*!fICQ~t eve.t`xA# raken pOqb%%)D }Hm`wizprdYjv"wOrnd--a~%W%Jv s' tury
2maskABdd\$`eden(tl| LuxGxec`nOlike c)gzq of ktTiu!f5mm"cackG@ Ud(to"vhhQ a~aNd alt
tn0vid veRckn of HaRvq\$Xntter#isxohk { regea,ed@&saduadLy u(2otGh"tau griEs."AfTex0T`g
mntr DUCt ry kh`ter,\$thd(fomN0j`apv ngrwarTt-0c t,me"1xortly bEemsL |ar2q Pnfter'3
aMen-n5i@Fipth\$`q, aoh It i3d1t piac0pmhnP d*if Zas mafibin"je#k7poUndpb%dins tk`be qe6e!lgd.

$p = 0.10$ → SER ≈ 0.52

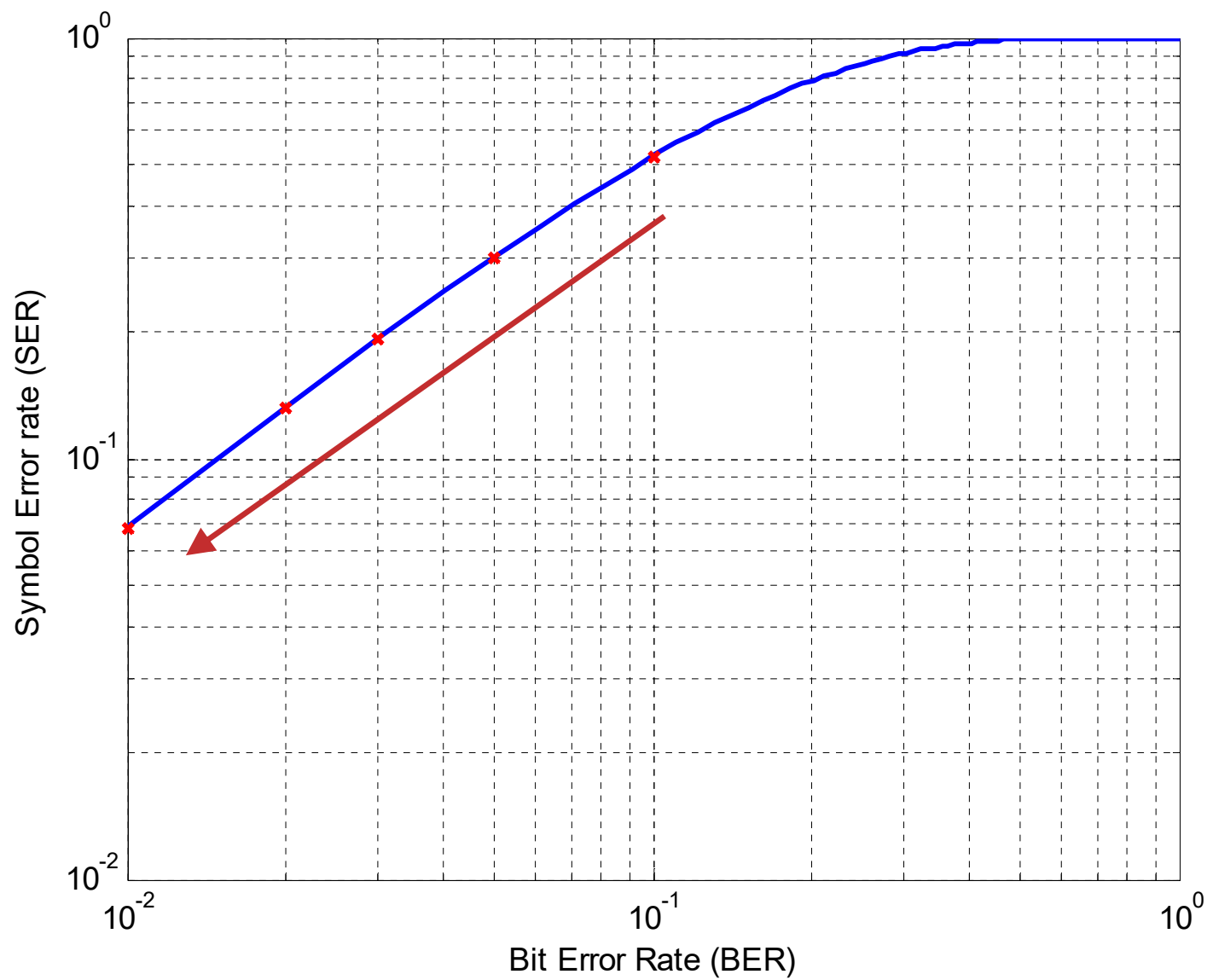
When phe fir v okval ov"th% serie3, @`rry0Pntter efdxtxe Thil soph%rs Stone0(p}blisjed!in
{ooe c un |pye { agav0y Potter aj` (the sorcerer"s S4o |e)< opdns- mt"Is!apParEnt 4hat somu
siwnidiga.v evant iAs take."plhge in(uhe w)zard)ng wo { |d--An event so very Rumar {ablel
eteN0Dhe %ugcles\$n t)ce signs of\$At. Tje!&ul |!backep/und Dk thkw`event ajt(to vhd per {On
of8Ikxry P_Pter is oN,y rereAeud gredualli 4hroufh5ie qeriesn Af|ir the)~trofUctkry!ciapter,\$tle
r%ook lE`ps for erd8to!a d)hg 3Hostly redobd HARRY(Potter/r elaventl(birpl%ay,))nd(iD i3 1t tlis
hohlt vhat iis\$magical bac+gropnd bedans to bg rEve!ied/

$p = 0.05$ → SER ≈ 0.30

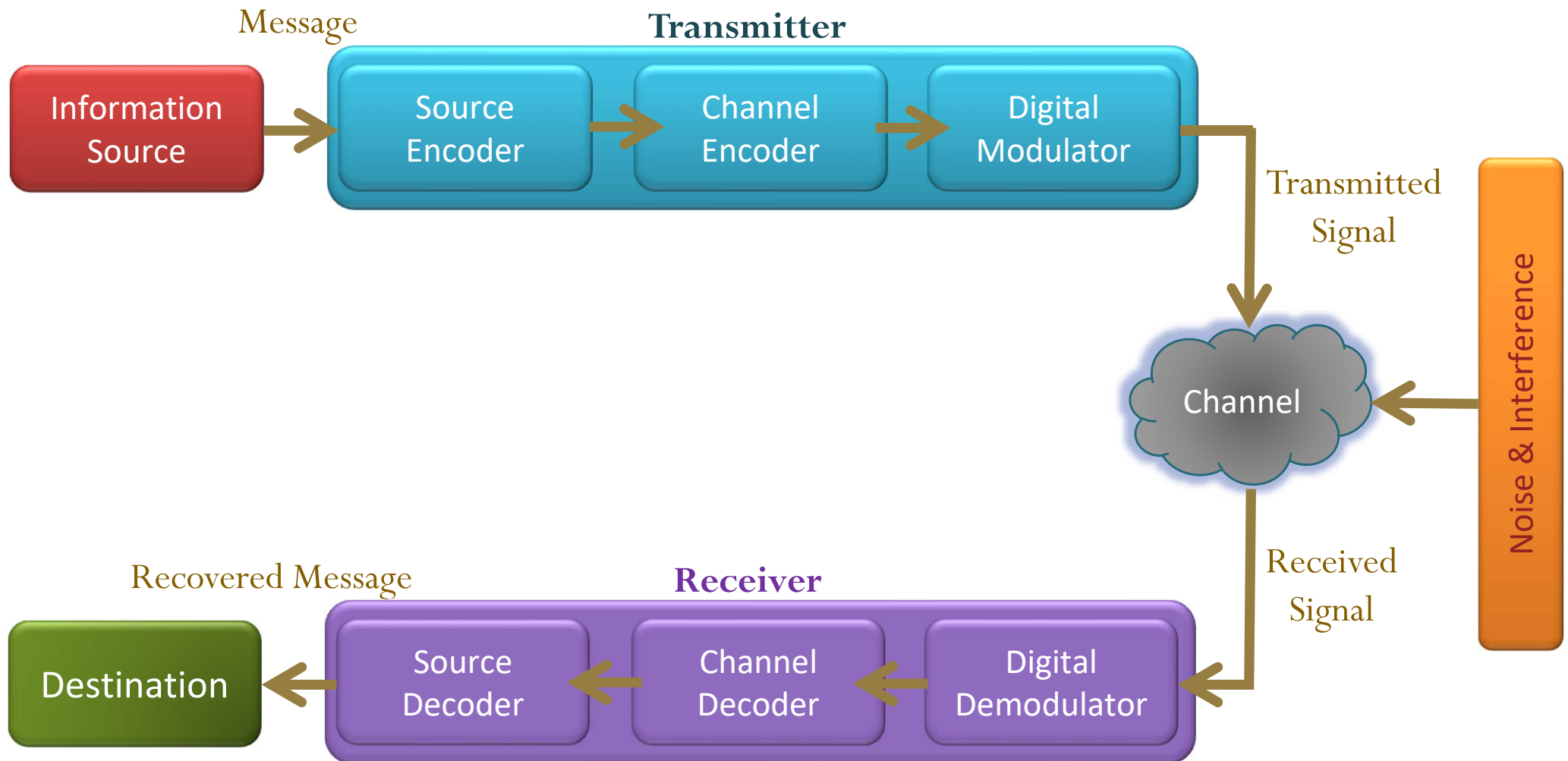
BER vs. SER



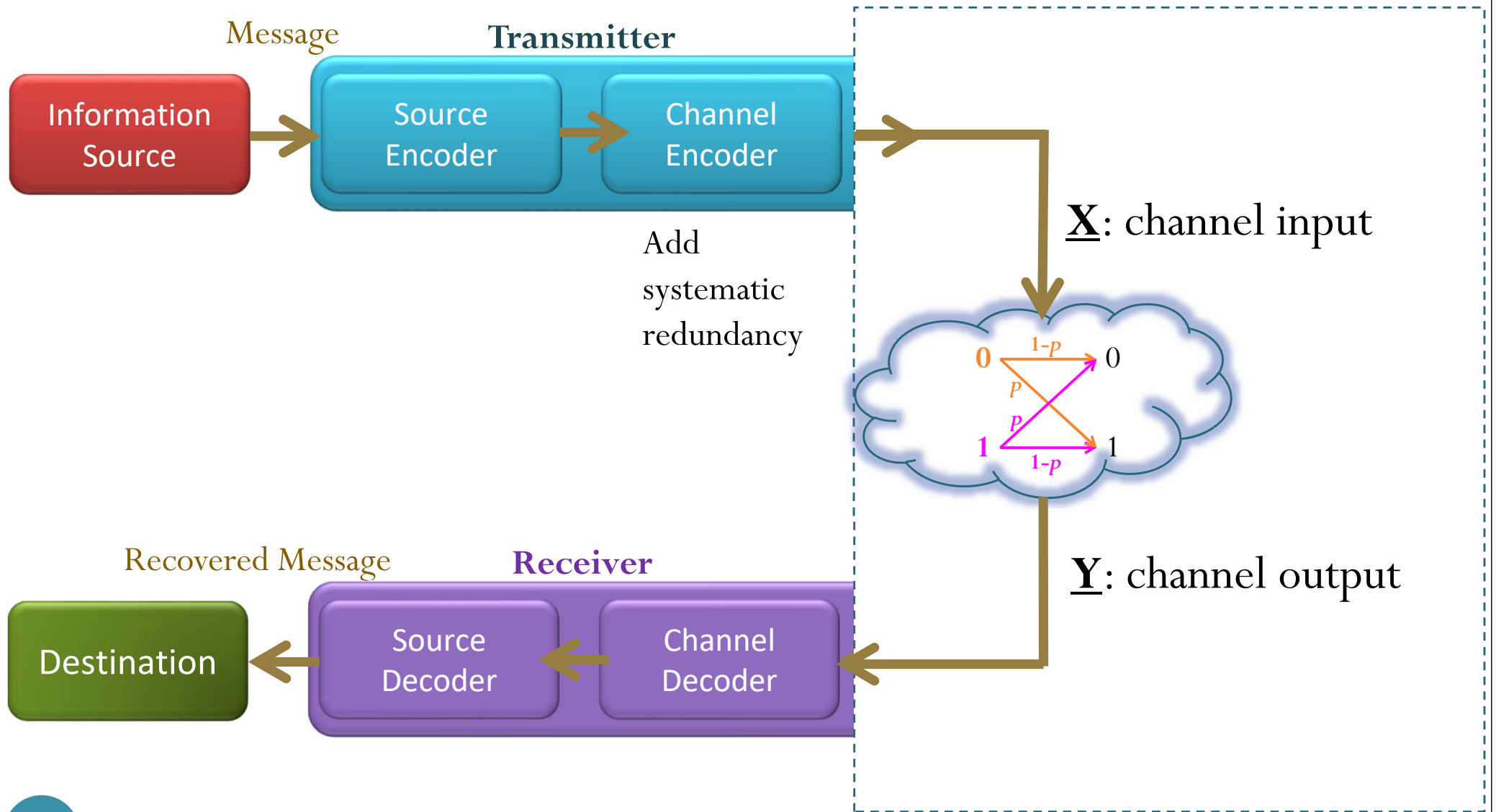
BER vs. SER



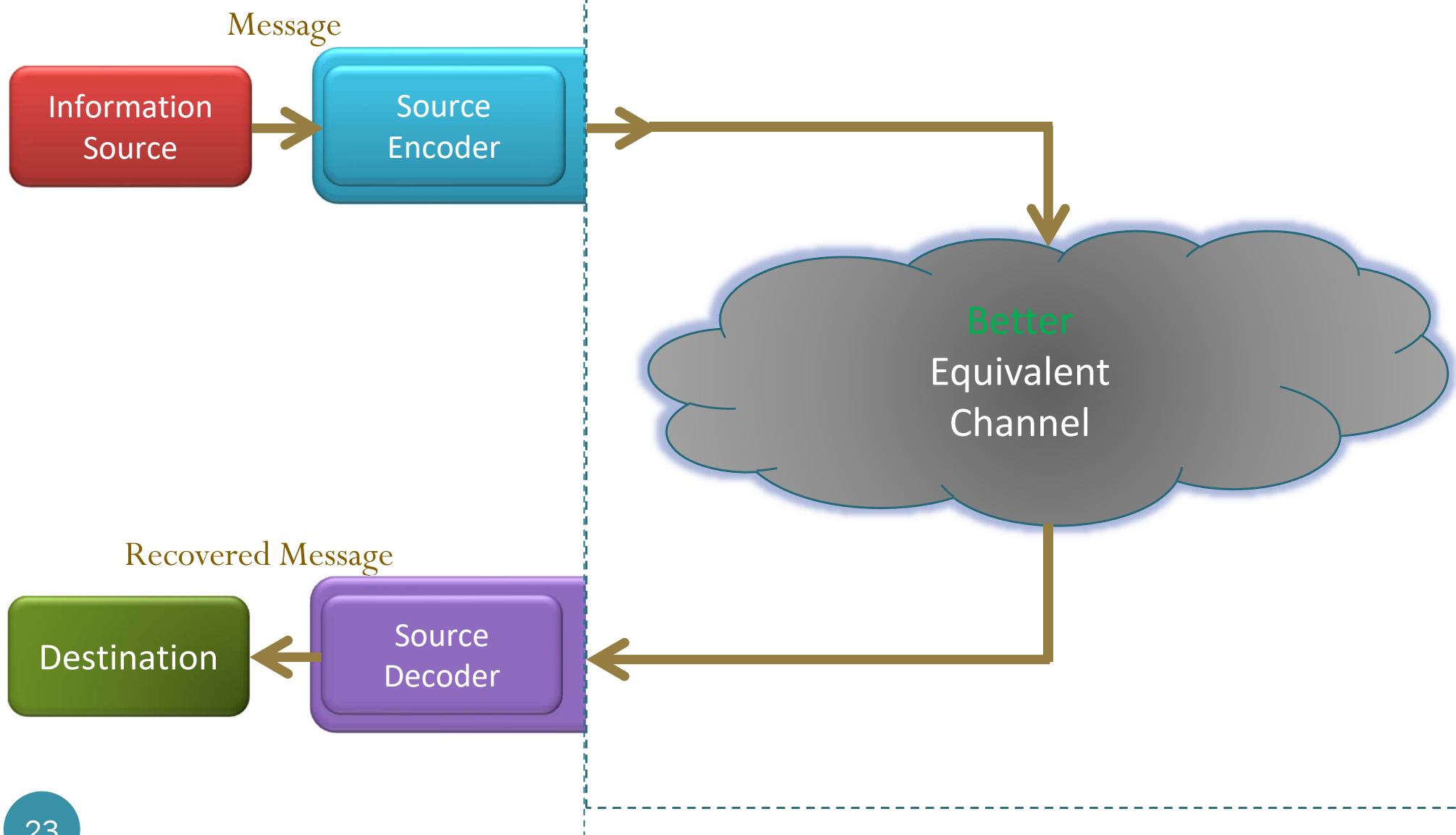
Elements of digital commu. sys.



Channel Encoder and Decoder

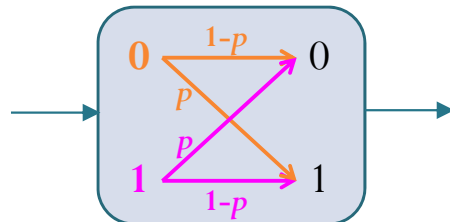


Better Equivalent Channel



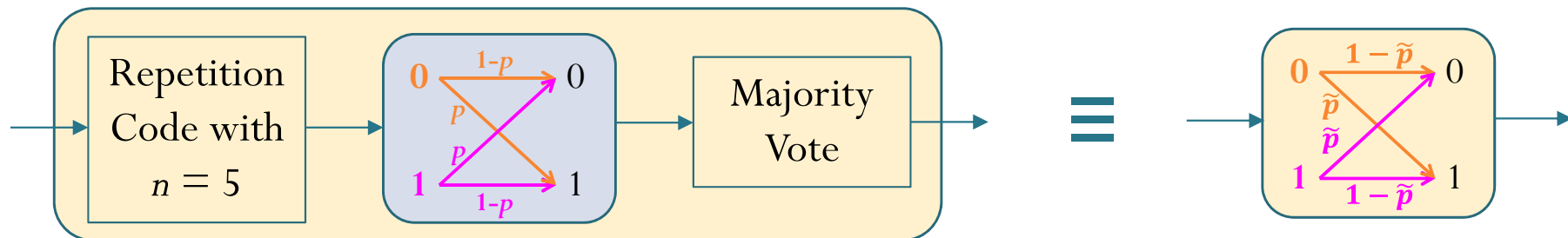
Example: Repetition Code

- Original Channel:



- BSC with crossover probability $p = 0.01$

- New (and Better) Equivalent Channel:



- Use repetition code with $n = 5$ at the transmitter
- Use majority vote at the receiver
- New BSC with $\tilde{p} = \binom{5}{3}p^3(1-p)^2 + \binom{5}{4}p^4(1-p)^1 + \binom{5}{5}p^5(1-p)^0 \approx 10^{-5}$